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# Structures of the $\mathrm{ReO}_{3}$-type with Recurrent Dislocations of Atoms: 'Homologous Series' of Molybdenum and Tungsten Oxides* 

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#### Abstract

The geometrical properties of structures of the $\mathrm{ReO}_{3}$-type showing recurrent dislocations of atoms, analogous to those present in $\mathrm{Mo}_{8} \mathrm{O}_{23}, \mathrm{Mo}_{9} \mathrm{O}_{26}$ and $\mathrm{W}_{20} \mathrm{O}_{58}$, are discussed. Two of the hypothetical atomic arrangements thus derived have been found to correspond to recently prepared molybdenum tungsten oxides ( $\mathrm{Mo}, \mathrm{W})_{10} \mathrm{O}_{29}$ and ( $\left.\mathrm{Mo}, \mathrm{W}\right)_{11} \mathrm{O}_{32}$. These compounds and the aforementioned molybdenum oxides are members of a 'homologous series', $\mathrm{Me}_{n} \mathrm{O}_{3 n-1}$, based on a common structural principle. There is almost complete agreement between the experimentally derived structures of these homologues and the corresponding ideal structures. The same is also true for $W_{20} \mathrm{O}_{58}$, the only hitherto known representative of the series $\mathrm{Me}_{n} \mathrm{O}_{3 n-2}$.


## Introduction

Investigations of molybdenum and tungsten oxides with an average valency of the metal atoms slightly less than six, namely $\mathrm{Mo}_{8} \mathrm{O}_{23}, \mathrm{Mo}_{9} \mathrm{O}_{26}$ and $\mathrm{W}_{20} \mathrm{O}_{58}$, have shown that the crystal structures of these compounds are mutually closely related and based on structures of the $\mathrm{ReO}_{3}$-type (Strukturbericht, 1937) containing dislocations of the atoms, suddenly occurring after long periods (Magnéli, 1948, 1950a, b). A geometrical discussion of hypothetical structures analogous to those derived for these oxides has proved to be valuable when investigating a number of double oxides of molybdenum and tungsten (Magnéli, 1952; Blomberg, Kihlborg \& Magnéli, 1953) and will be described here. A comparison will be given between the crystal structures of the known oxides of this type and the corresponding hypothetical structures.

## Structural principles of $\mathrm{Mo}_{8} \mathrm{O}_{\mathbf{2 3}}, \mathrm{Mo}_{9} \mathrm{O}_{\mathbf{2 6}}$ and $\mathrm{W}_{20} \mathrm{O}_{58}$

The crystal structures of these oxides may be described as built up of metal-oxygen octahedra joined by corners to form blocks of the $\mathrm{ReO}_{3}$-type, which extend infinitely through the crystals in two dimensions and have a finite, characteristic width in a third direction. The blocks are mutually joined along folded planes by

[^0]octahedra having edges in common. These planes correspond to the recurrent dislocations of atoms occurring in the $\mathrm{ReO}_{3}$-type structure.

The structures of $\mathrm{Mo}_{8} \mathrm{O}_{23}$ and $\mathrm{Mo}_{9} \mathrm{O}_{26}$ differ only in respect of the finite width of the $\mathrm{ReO}_{3}$-type blocks, which extend through eight and nine $\mathrm{MoO}_{6}$ octahedra respectively. The appearance of the planes joining the blocks is the same in both oxides and is characterized by the recurrent occurrence of groups of four $\mathrm{MoO}_{6}$ octahedra joined by sharing edges (Figs. l(d), 4(a), (b)). There is a regular, alternating displacement of the metal atoms in the octahedra, parallel to the $b$ axes of the monoclinic unit cells (normal to the plane of the figures). The metal atoms thus form puckered layers parallel to the ac plane. There is a marked lengthening of the $\mathrm{MoO}_{6}$ octahedra perpendicularly to this plane. The molybdenum atoms of the quadruplets of octahedra sharing edges are situated in planes characteristically tilted. The considerable distortion of the $\mathrm{MoO}_{6}$ octahedra may suggest that the molybdenum atoms can be considered as co-ordinating five oxygen atoms in a square-pyramidal arrangement. For the following discussion, however, the simplified picture of distorted $\mathrm{MoO}_{6}$ octahedra is more convenient.

The dislocation planes occurring in $\mathrm{W}_{20} \mathrm{O}_{58}$ are of a somewhat different appearance from those present in the molybdenum oxides, in showing groups of six metal-oxygen octahedra joined by sharing edges (Fig. 1(e)). The finite extension of the $\mathrm{ReO}_{3}$-type blocks in this structure amounts to twenty octahedra. The metal atom layers are not puckered.


Fig. 1. (a) The $\mathrm{ReO}_{3}$-type structure. The Me atoms (small, black circles) are co-ordinated with six oxygen atoms (large circles) to form $\mathrm{MeO}_{6}$ octahedra, which are joined by sharing corners. The oxygen atoms nearest to the observer are omitted to show the underlying metal atoms. The extension of one unit cell is indicated. (b) The border region of the $\mathrm{ReO}_{3}$ type blocks of structures $\mathrm{Me}_{n} \mathrm{O}_{3 n}(m=1)$, built up of regular $\mathrm{MeO}_{6}$ octahedra. The projection of the contour plane of the blocks is indicated by a broken line and the direction of characteristic finite extension of the blocks by an arrow. (c) The border region of structures $\mathrm{Me}_{n} \mathrm{O}_{3 n-1}(m=2)$ with non-puckered metal atom layers. (d) The corresponding arrangement for lattices $\mathrm{Me}_{n} \mathrm{O}_{3 n-1}$ with metal-atom layers puckered in the same way as those of $\mathrm{Mo}_{8} \mathrm{O}_{23}$ and $\mathrm{Mo}_{9} \mathrm{O}_{26}$. The displacements of the metal atoms are indicated by the vertical and horizontal shading of the small circles. (e) The border region of structures $\mathrm{Me}_{n} \mathrm{O}_{3 n-2}(m=3)$.

## Hypothetical structures

It is obvious that a large number of structures can be formed by combining metal-oxygen octahedra ( $\mathrm{MeO}_{6}$ octahedra) in a similar manner to those present in the molybdenum and tungsten oxides described in the preceding paragraphs. However, the investigation was essentially restricted to discussions of the effects in atomic arrangements analogous to that of $\mathrm{W}_{20} \mathrm{O}_{58}$ of varying ( $a$ ) the finite width of the $\mathrm{ReO}_{3}$-type blocks ( $n \mathrm{MeO}_{6}$ octahedra) and (b) the number of pairs of octahedra participating in the mutual joining of these blocks ( $m$ pairs of $\mathrm{MeO}_{6}$ octahedra). Moreover, the geometrical properties of 'puckered' structures analogous to those of $\mathrm{Mo}_{8} \mathrm{O}_{23}$ and $\mathrm{Mo}_{9} \mathrm{O}_{26}(m=2)$ were studied as a function of $n$. The value of $m$ was con-
sidered to be the fundamental variable as it determines the shape of the border planes of the $\mathrm{ReO}_{3}$-type blocks and defines two of the lattice constants of the structures (parallel to this plane). It was thus natural to classify all the structures considered according to $m$ in groups or series, the various members of which differ only in respect of the finite width of the blocks. It seems reasonable to characterize these series, expressible by general formulae and built on common structural principles, as homologous series. In the following the geometrical properties of some of these series will be discussed.

## The $\mathrm{Me}_{n} \mathrm{O}_{3 n}$ series

When the rearrangement of the $\mathrm{ReO}_{3}$-type structure is restricted to the introduction of groups of two $\mathrm{MeO}_{6}$ octahedra sharing edges $(m=1)$, as illustrated in Fig. $1(b)$, every oxygen atom will still belong to two octahedra and the ratio of metal atoms to oxygen atoms will remain 1:3. The unit-cell content will thus correspond to the formula $\mathrm{Me}_{n} \mathrm{O}_{3 n}(n \geq 2)$. The symmetry of the arrangement will be $P 2 / m$ for $n$ even and $P m m m$ for $n$ odd. No representatives of this series are known and as it is to some extent divergent from the other ones, it will be disregarded in what follows.

## The $\mathrm{Me}_{n} \mathrm{O}_{3 n-1}$ series

(a) Non-puckered metal atom layers.-The appearance of the border regions when four $\mathrm{MeO}_{6}$ octahedra are participating in the joining of the blocks ( $m=2$ ) is illustrated in Fig. 1(c). Two of the oxygen atoms in these groups belong to three octahedra, while all the other oxygen atoms of the structure are common to two octahedra. The general formula of the series will thus be $\mathrm{Me}_{n} \mathrm{O}_{3 n-1}(n \geq 4)$. The symmetry of the arrangement is $P 2 / m$.
(b) Puckered metal atom layers.-If the metal atoms are assumed to form puckered layers (Fig. l(d)), as in $\mathrm{Mo}_{8} \mathrm{O}_{23}$ and $\mathrm{Mo}_{9} \mathrm{O}_{26}$, this will change the symmetry of the structure to $P 2 / a$ and double the unit cell, and thus give the cell content $2 \mathrm{Me}_{n} \mathrm{O}_{3 n-1}$. Some doubt may exist which minimum value of $n$ ought to be assigned to such an atomic arrangement.

## General series

If groups of $2 m \mathrm{MeO}_{6}$ octahedra with edges in common ( $m$ pairs of octahedra) are taking part in the joining of the blocks (cf. Fig. l(e), where $m=3$ ) structures of the composition $\mathrm{Me}_{n} \mathrm{O}_{3 n-m+1}$ and of symmetry $P 2 / m$ will result (unpuckered metal atom layers).

When calculating unit-cell dimensions and atomic positions of the various structures, combinations of regular $\mathrm{MeO}_{6}$ octahedra may be considered, as was actually done in some previous articles (Magnéli, 1948, 1952). This, however, represents a rather rough approximation, as the close approach of the high-valent metal atoms of the octahedra joined by sharing edges
must imply considerable repulsive forces between these atoms, leading to lengthening of the interatomic distances and a corresponding distortion of the $\mathrm{MeO}_{6}$ octahedra involved. The various parameters of the hypothetical structures were thus calculated assuming (a) that the metal-atom positions within the blocks are arranged as in an ideal $\mathrm{ReO}_{3}$-type structure with the interatomic spacing equal to $d$ (or occasionally rearranged to form puckered layers) and (b) that the distances between the metal atoms of octahedra joined by edges is $e$, exceeding the distance of $d / / / 2$ valid for regular octahedra. In the calculations the latter condition was introduced by a parameter $\delta$ (cf. Fig. 2)


Fig. 2. The appearance of the border regions of the $\mathrm{ReO}_{3}$-type blocks when the repulsion between metal atoms of $\mathrm{MeO}_{6}$ octahedra sharing edges is taken into consideration (black metal atom circles). This arrangement corresponds to a displacement of all the metal atoms of the right block by $-(d / V 2) \delta$ (in the $\xi$-direction) and by $(d / / 2) \delta$ (in the $\zeta$ direction) from the atomic positions corresponding to regular $\mathrm{MeO}_{6}$ octahedra (open circles). The oxygen atoms are omitted.
related to $d$ and $e$ by the equation

$$
\delta=V\left\{(e / d)^{2}-\frac{1}{4}\right\}-\frac{1}{2}
$$

In order to simplify the mathematical expressions a few auxiliary variables were introduced. The finite width of the blocks, $n$, was thus formulated

$$
n=(m+1) p+q
$$

where $p$ and $q$ are integers and the latter has values $0,1,2,3, \ldots, m$ for unpuckered structures and $-1,0,1$ for puckered structures of the series $\mathrm{Me}_{n} \mathrm{O}_{3 n-1}$. Furthermore, the monoclinic angle $\beta$ was expressed by

$$
\beta=\beta^{\prime}+\beta^{\prime \prime}
$$

where $\beta^{\prime}$ and $\beta^{\prime \prime}$ represent the slopes of the $c$ and $a$ axes against the diagonal of the basic $\mathrm{ReO}_{3}$-type mesh, as indicated in Fig. 2. The value of $\beta^{\prime \prime}$ is thus independent of $n$.

The geometrical characteristics, also including the metal-atom positions, for unpuckered structures with $m \geq 2$ are summarized in Table 1 and the corresponding data for puckered structures of the series $\mathrm{Me}_{n} \mathrm{O}_{3 n-1}$ in Table 2. General expressions for the oxygen positions may of course also be derived. It was, however, not thought worth while to undertake this somewhat tedious work as the oxygen co-ordinates as obtained

Table 1. Geometrical characteristics of non-puckered structures $\mathrm{Me}_{n} \mathrm{O}_{3 n-m+1}(m \geq 2 ; n=p(m+1)+q$,

$$
q=0,1,2, \ldots, m)
$$

Symmetry: $P 2 / m$.
Unit-cell content: $\mathrm{Me}_{n} \mathrm{O}_{3 n-m+1}$.
Unit-cell dimensions:

$$
\begin{aligned}
& a=d V\left(m^{2}+1\right), \\
& b=d \text {, } \\
& c=(d / \sqrt{ }) \vee\left\{[n-(m-1)(p+1)+\delta]^{2}\right. \\
& \left.+[(n+1)-(m+1)(p+1)-\delta]^{2}\right\} ; \\
& \beta=\beta^{\prime}+\beta^{\prime \prime} ; \beta^{\prime}=\operatorname{arctg}\left[\frac{(m+1)(p+1)-(n+1)+\delta}{n-(m-1)(p+1)+\delta}\right], \\
& \beta^{\prime \prime}=\operatorname{arctg}\left[\frac{m+1}{m-1}\right] .
\end{aligned}
$$

Metal atom positions $1(a), \mathrm{l}(e)$, or $2(n)$ :

$$
\begin{aligned}
& x_{i}=j \frac{d \sin \left(\beta^{\prime}+45^{\circ}\right)}{2 a \sin \left(180^{\circ}-\beta\right)}+\frac{k}{2} \\
& y_{i}=\frac{1}{2} \\
& z_{i}=j \frac{d \sin \left(\beta^{\prime \prime}-45^{\circ}\right)}{2 c \sin \left(180^{\circ}-\beta\right)}
\end{aligned}
$$

where

$$
j= \begin{cases}0,2,4, \ldots,(n-1) & \text { for } n \text { odd } \\ 1,3,5, \ldots,(n-1) & \text { for } n \text { even }\end{cases}
$$

and

$$
k=\left\{\begin{aligned}
& 1 \text { for } n<2(m+1) \\
& 3(m+1) \leq n<4(m+1) \\
& 5(m+1) \leq n<6(m+1) \text { etc. } \\
& \\
& 0 \text { for } 2(m+1) \leq n<3(m+1) \\
& 4(m+1) \leq n<5(m+1) \text { etc. }
\end{aligned}\right.
$$

Table 2. Geometrical characteristics of puckered structures

$$
\mathrm{Me}_{n} \mathrm{O}_{3 n-1}(m=2 ; n=3 p+q, q=-1,0,+1)
$$

Symmetry: P2/a.
Unit cell content: $2 \mathrm{Me}_{n} \mathrm{O}_{3 n-1}$.
Unit cell dimensions:
$a=2 d \downarrow 5$,
$b>d$,
$c=(d / \sqrt{ } 2) \vee\left\{[2 p-1+\delta]^{2}+[6 p-2 n-2-\delta]^{2}\right\} ;$
$\beta=\beta^{\prime}+\beta^{\prime \prime} ; \beta^{\prime}=\operatorname{arctg}\left[\frac{2 n+2-6 p+\delta}{2 p-1+\delta}\right], \beta^{\prime \prime}=\operatorname{arctg}\left[\frac{m+1}{m-1}\right]$.
Metal atom positions $2(e)$ or $4(g)$ :

$$
\begin{aligned}
& x_{i}=j \frac{d \sin \left(\beta^{\prime}+45^{\circ}\right)}{2 a \sin \left(180^{\circ}-\beta\right)}+\frac{k}{4}, \\
& y_{i}=\frac{1}{2}+l v, \\
& z_{i}=j \frac{d \sin \left(\beta^{\prime \prime}-45^{\circ}\right)}{2 c \sin \left(180^{\circ}-\beta\right)}
\end{aligned}
$$

where

$$
\begin{aligned}
j & =\left\{\begin{array}{l}
0,2,4, \ldots,(n-1) \text { for } n \text { odd } \\
1,3,5, \ldots,(n-1) \text { for } n \text { even }
\end{array}\right. \\
k & =\left\{\begin{array}{l}
0 \text { for } n \text { even } \\
1 \text { for } n \text { odd }
\end{array}\right. \\
l & =\left\{\begin{array}{l}
+1 \text { for } i=i_{\text {max }}, i_{\max }-1, i_{\max }-3, i_{\max .}-5, \ldots \\
-1 \text { for } i=i_{\max }-2, i_{\max }-4, i_{\max .}-6, \ldots
\end{array}\right.
\end{aligned}
$$

and $v$ is a small positive quantity $(\sim 0.1)$.
from experimental data must be relatively inaccurate and thus not very well fitted for comparisons with the ideal positions. Minor modifications (making $\delta$ equal to zero in the direction of the $\xi$ axis of Fig. 2) will
make the expressions of Table 1 also valid for lattices of the series $\mathrm{Me}_{n} \mathrm{O}_{3 n}(m=1)$.

For reasons of lattice symmetry, other values of $\beta$ and $c$ have to be utilized for puckered structures $\mathrm{Me}_{n} \mathrm{O}_{3 n-1} n \equiv 1(\bmod .3)$ than those chosen for the corresponding unpuckered structures. This is also the background for the use of different sets of values for $q$ in the expression $n=p(m+1)+q$ for unpuckered and puckered structures (vide supra).

## An ideal $\mathrm{V}_{2} \mathrm{O}_{5}$-type structure

An interesting formal relation exists between structures of high values of $m$ and that of vanadium pentoxide, which may be considered as built up of infinitely extending groups of $\mathrm{MeO}_{6}$ octahedra joined by edges $(m=\infty)$. For regular octahedra (cf. Fig. 3) this ar-


Fig. 3. The $\mathrm{Me}_{2} \mathrm{O}_{5}$ structure built up of regular $\mathrm{MeO}_{6}$ octahedra. The unit mesh corresponds to the ac projection of an idealized $\mathrm{V}_{2} \mathrm{O}_{5}$-type structure, $a$ being the longer axis.
rangement would correspond to an orthorhombic unit cell with the dimensions $a=3 d, b=c=d$ ( $d$ being the length of the space diagonal of the octahedron). The actual structure of vanadium pentoxide (Byström, Wilhelmi \& Brotzen, 1950) shows a considerable deviation from this idealized structure, the $\mathrm{VO}_{6}$ octahedra being highly distorted towards square $\mathrm{VO}_{5}$ pyramids with an additional remote oxygen atom. The distortion is reflected by the axial ratio of the orthorhombic unit cell being $a: b: c=3 \cdot 07: 1 \cdot 17: 0.95$ (referred to a plausible value of $d=3.75 \AA$ ). However, the vanadium pentoxide structure is able to dissolve considerable amounts of molybdenum trioxide, which modifies the structure towards a more regular octahedral arrangement (Magnéli, Oughton \& Blomberg, 1951). The axial ratio for a solid solution with $14 \%$ of the vanadium(V)atoms substituted by molybdenum(VI)atoms (and the proper amount of vacant metal-atom sites) is $a: b: c=3 \cdot 10: 1 \cdot 14: 0 \cdot 96$.

## X-ray patterns

A common feature of the X-ray diffraction patterns of the structures derived above will be the appearance of a sub-structure effect caused by the dominating influence of the $\mathrm{ReO}_{3}$-type arrangement of the atoms. Large finite extension of the blocks, regularity of the


Fig. 4. The observed crystal structures of $(a) \mathrm{Mo}_{8} \mathrm{O}_{23},(b) \mathrm{Mo}_{9} \mathrm{O}_{26}$, (c) $(\mathrm{Mo}, \mathrm{W})_{10} \mathrm{O}_{29}$ and (d) $(\mathrm{Mo}, \mathrm{W})_{11} \mathrm{O}_{32}$.

Table 3. Comparison between observed and ideal unit-cell dimensions for oxides $\mathrm{Me}_{n} \mathrm{O}_{3 n-1}$ and $\mathrm{Me}_{n} \mathrm{O}_{3 n-2}$

| $\quad$ Oxide |
| :--- |
| $\mathrm{Mo}_{8} \mathrm{O}_{23}$ |
| $\mathrm{Mo}_{9} \mathrm{O}_{26}$ |
| $(\mathrm{Mo}, \mathrm{W})_{10} \mathrm{O}_{29}$ |
| $(\mathrm{Mo}, \mathrm{W})_{11} \mathrm{O}_{32}$ |
| $\mathrm{~W}_{20} \mathrm{O}_{58}$ |

Observed unit cell dimensions

|  | $\overbrace{a(\AA)}$ | $b(\AA)$ | $c(\AA)$ | $\beta$ |
| :--- | :--- | :--- | :--- | :--- |
| 2 | 16.90 | 4.055 | 13.38 | $73^{\circ} 73$ |
| 2 | 16.74 | 4.019 | 14.53 | $95^{\circ} 45$ |
| 2 | 17.0 | 4.00 | 17.5 | $111^{\circ}$ |
| 2 | 16.6 | 4.00 | 18.7 | $74^{\circ}$ |
| 3 | 12.05 | 3.767 | 23.59 | $85^{\circ} 28$ |

Ideal unit cell dimensions

| $a(\AA)$ | $b(\AA)$ | $c(\AA)$ | $\beta$ |
| :--- | ---: | ---: | ---: |
| 16.77 | $>3.75$ | 13.81 | $73^{\circ} 91$ |
| 16.77 | $>3.75$ | 15.00 | $94^{\circ} 58$ |
| 16.77 | $>3.75$ | 17.76 | $110^{\circ} 47$ |
| 16.77 | $>3.75$ | 19.12 | $73^{\circ} 23$ |
|  |  |  |  |
| 11.86 | 3.75 | 23.37 | $84^{\circ} 75$ |

Table 4. Comparison between observed and ideal atomic co-ordinates for oxides $\mathrm{Me}_{n} \mathrm{O}_{3 n-1}$ Space-group P2/a. (The co-ordinate values refer to sequences of atoms in the directions of the arrows of Fig. 4.)

| Oxide | Me atom | Point position | Observed co-ordinates |  |  | Ideal co-ordinates |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $x$ | $y$ | $z$ | $x$ - | $y$ | $z$ |
| $\mathrm{Mo}_{8} \mathrm{O}_{23}$ | $\mathrm{Mo}_{1}$ | 4(g) : | 0.085 | 0.59 | 0.063 | 0.085 | $>\frac{1}{2}$ | 0.063 |
|  | $\mathrm{MO}_{2}$ | $4(g)$ : | $0 \cdot 254$ | $0 \cdot 41$ | $0 \cdot 188$ | 0.257 | $<\frac{1}{2}$ | $0 \cdot 190$ |
|  | $\mathrm{Mo}_{3}$ | 4(g) : | $0 \cdot 423$ | 0.59 | 0.316 | $0 \cdot 428$ | $>\frac{1}{2}$ | 0.316 |
|  | $\mathrm{Mo}_{4}$ | 4(g): | 0.596 | 0.59 | $0 \cdot 445$ | 0.599 | $>\frac{1}{2}$ | $0 \cdot 442$ |
| $\mathrm{Mo}_{9} \mathrm{O}_{26}$ | $\mathrm{Mo}_{1}$ | 2(e): | 4 | 0.41 | 0 | $\ddagger$ | $<\frac{1}{2}$ | 0 |
|  | $\mathrm{Mo}_{2}$ | 4(g) : | $0 \cdot 456$ | 0.59 | $0 \cdot 113$ | $0 \cdot 458$ | $>\frac{1}{2}$ | $0 \cdot 112$ |
|  | $\mathrm{Mo}_{3}$ | $4(\mathrm{~g})$ : | 0.662 | $0 \cdot 41$ | $0 \cdot 221$ | 0.666 | $<\frac{1}{2}$ | 0.224 |
|  | $\mathrm{MO}_{4}$ | $4(\mathrm{~g})$ : | 0.869 | 0.59 | 0.334 | 0.874 | $>\frac{1}{2}$ | 0.337 |
|  | $\mathrm{Mo}_{5}$ | 4(g) : | 1.082 | 0.59 | 0.450 | 1.082 | $>\frac{1}{2}$ | $0 \cdot 449$ |
| $(\mathrm{Mo}, \mathrm{W})_{10} \mathrm{O}_{29}$ | $\mathrm{Me}_{1}$ | 4(g) : | $0 \cdot 117$ | 0.42 | 0.050 | $0 \cdot 119$ | $<\frac{1}{2}$ | 0.050 |
|  | $\mathrm{Me}_{2}$ | 4(g): | 0.353 | 0.58 | $0 \cdot 150$ | 0.356 | $>\frac{1}{2}$ | 0.151 |
|  | $\mathrm{Me}_{3}$ | $4(\mathrm{~g})$ : | 0.591 | 0.42 | $0 \cdot 252$ | 0.594 | $<\frac{1}{2}$ | 0.252 |
|  | $\mathrm{Me}_{4}$ | $4(g)$ : | 0.825 | 0.58 | 0.352 | 0.832 | $>\frac{1}{2}$ | 0.352 |
|  | $\mathrm{Me}_{5}$ | $4(\mathrm{~g})$ : | 1.065 | 0.58 | 0.457 | 1.069 | $>\frac{1}{2}$ | 0.453 |
| $(\mathrm{Mo}, \mathrm{W})_{11} \mathrm{O}_{32}$ | $\mathrm{Me}_{1}$ | $2(e)$ : | 1 | 0.60 | 0 | $\stackrel{1}{1}$ | $>\frac{1}{2}$ | 0 |
|  | $\mathrm{Me}_{2}$ | $4(g)$ : | 0.418 | 0.40 | 0.092 | $0 \cdot 420$ | $<\frac{1}{2}$ | 0.092 |
|  | $\mathrm{Me}_{3}$ | 4(g) : | 0.588 | 0.60 | $0 \cdot 183$ | 0.590 | $>\frac{1}{2}$ | $0 \cdot 183$ |
|  | $\mathrm{Me}_{4}$ | 4(g): | 0.757 | $0 \cdot 40$ | $0 \cdot 277$ | 0.759 | $<\frac{1}{2}$ | 0.275 |
|  | $\mathrm{Me}_{5}$ | $4(g)$ : | 0.927 | $0 \cdot 60$ | 0.370 | 0.929 | $>\frac{1}{2}$ | 0.366 |
|  | Me ${ }_{6}$ | 4(g) : | 1.096 | $0 \cdot 60$ | $0 \cdot 458$ | 1.099 | $>\frac{1}{2}$ | $0 \cdot 458$ |

$\mathrm{MeO}_{6}$ octahedra and high scattering power of the metal atoms in comparison with that of oxygen will cooperate to give rise to single-crystal photographs with all strong reflexions concentrated in multiplets close to the reciprocal-lattice points of a primitive cubic sub-structure with the cell constant $a^{\prime}=d$ (the space diagonal of the regular $\mathrm{MeO}_{6}$ octahedron). In extreme cases all the interjacent reflexions may be too weak to be detectable in the X-ray patterns (Magnéli, 1951). If the metal-atom layers are puckered, the sub-structure will be tetragonal with $a^{\prime}=d / 2$ and $c^{\prime}$ slightly larger than $d$.

## Comparison between observed and hypothetical structures

One of the essential interests in the considerations presented above is of course whether it is possible to prepare new chemical compounds with structures corresponding to these hypothetical atomic arrangements. Investigations of mixed molybdenum tungsten oxides recently carried out at this Institute have given an affirmative answer to this question. Thus, structure analyses of two such phases carried out on the basis of
the geometrical discussions described above demonstrated the existence of the compounds ( $\mathrm{Mo}, \mathrm{W})_{10} \mathrm{O}_{29}$ and ( $\mathrm{Mo}, \mathrm{W})_{11} \mathrm{O}_{32}$, corresponding to members of the series $\mathrm{Me}_{n} \mathrm{O}_{3 n-1}$ with puckered metal-atom layers (Blomberg, Kihlborg \& Magnéli, 1953). The crystal structures of the four homologues known at present are illustrated in Fig. 4. Further molybdenum tungsten oxide phases have been isolated and their X-ray patterns suggest that they are still higher members of this series. However, this matter has not so far been investigated in detail and merits further studies. Attempts to prepare compounds belonging to series other than $\mathrm{Me}_{n} \mathrm{O}_{3 n-1}$ are also desirable, $\mathrm{W}_{20} \mathrm{O}_{58}$ at present being the only known substance of this kind. It is obvious, however, that the identification of structures containing very wide blocks must be a rather intricate task, owing to the peculiar character of their X-ray patterns.

It is also of considerable interest to correlate the geometrical properties of the actually known structures with those of the hypothetical atomic arrangements. In this way it should be possible to get an idea of the feasibility of the arguments put forward in this article in predicting the properties of unknown sub-

Table 5. Comparison between observed and ideal atomic co-ordinates for $\mathrm{W}_{20} \mathrm{O}_{58}$
Space-group $\operatorname{P2} / m$

| W atom | Point position | Observed co-ordinates |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | $x$ | $y$ | $z$ |
| $\mathrm{W}_{1}$ | $2(n)$ : | 0.647 | $\frac{1}{2}$ | 0.026 |
| $\mathrm{W}_{2}$ | $2(n)$ : | 0.937 | $\frac{1}{2}$ | 0.077 |
| $\mathrm{W}_{3}$ | 2(n): | $0 \cdot 227$ | $\frac{1}{2}$ | 0.128 |
| $\mathrm{W}_{4}$ | $2(n)$ : | 0.519 | $\frac{1}{2}$ | $0 \cdot 178$ |
| $\mathrm{W}_{5}$ | 2(n): | $0 \cdot 808$ | $\frac{1}{2}$ | $0 \cdot 229$ |
| $\mathrm{W}_{6}$ | $2(n)$ : | 0.097 | $\frac{1}{2}$ | 0.280 |
| $\mathrm{W}_{7}$ | $2(n)$ : | $0 \cdot 390$ | $\frac{1}{2}$ | $0 \cdot 331$ |
| $\mathrm{W}_{8}$ | $2(n)$ : | $0 \cdot 678$ | $\frac{1}{2}$ | 0.382 |
| W | $2(n)$ : | 0.970 | $\frac{1}{2}$ | $0 \cdot 432$ |
| $\mathrm{W}_{10}$ | $2(n)$ : | $0 \cdot 260$ | $\frac{1}{2}$ | $0 \cdot 483$ |

$\overbrace{x}^{c}$ Ideal co-ordinates
stances of this type which might exist. When calculating the data for the ideal structures a value of $d=3.75 \AA$ was chosen, which was supposed to be a reasonable value for the distance between the metal atoms of regular $\mathrm{MeO}_{6}$ octahedra sharing corners. The distance between the metal atoms of octahedra joined by edges (e) was found to be $3 \cdot 25-3 \cdot 3 \AA$ in $\mathrm{Mo}_{8} \mathrm{O}_{23}$, $\mathrm{Mo}_{9} \mathrm{O}_{26}$ and $\mathrm{W}_{20} \mathrm{O}_{58}$, and this value, corresponding to $\delta=0 \cdot 21$, was adopted here.

The observed and ideal unit-cell dimensions of the four known members of the series $\mathrm{Me}_{n} \mathrm{O}_{3 n-1}$, and of $\mathrm{W}_{20} \mathrm{O}_{58}$, are listed in Table 3. The data for the simple oxides are obtained from powder photographs and those for the mixed oxides, the molybdenum:tungsten ratios of which are not accurately known, from singlecrystal investigations. The deviations obtained for the monoclinic angles are throughout less than $1^{\circ}$. Comparisons of the lengths of the $a$ and $c$ axes will of course suffer from the inaccuracy of the value assigned to $d$. However, the deviations do not exceed $1.6 \%$ for the $a$ axes. For the $\mathrm{Me}_{n} \mathrm{O}_{3 n-1}$ compounds, the observed $c$ axes are throughout somewhat shorter (less than $3 \cdot 1 \%$ ) than the ideal ones. The observed $a$ and $c$ axes for $\mathrm{W}_{20} \mathrm{O}_{58}$. are both a little longer than the ideal values. Introducing a $d$ value equal to the observed length of the $b$ axis of this substance $(3.767 \AA$ instead of $3.75 \AA$ ) will reduce this small deviation by giving ideal dimensions of $11.91 \AA$ and $23.48 \AA$ respectively.

A comparison between the structural details of the observed and ideal structures may be made from the metal-atom positions given in Table 4 (for $\mathrm{Me}_{n} \mathrm{O}_{3 n-1}$ ) and Table 5 (for $\mathrm{W}_{20} \mathrm{O}_{58}$ ). As the inaccuracy of the experimentally derived $x$ and $z$ co-ordinates is likely
to amount to a few per mille of the axial lengths, the agreement must be considered to be almost perfect for all the compounds.

The close agreement thus found to exist between the observed and hypothetical structures evidently fully justifies the assumptions concerning the atomic arrangement within the blocks and at their joint planes made when deriving the geometrical properties of the latter. The expressions given in Table 1 and 2 are also likely to be capable of predicting the detailed structural arrangement of at present unknown members of these series. They have already proved to be of value when deriving the crystal structures of $(\mathrm{Mo}, \mathrm{W})_{10} \mathrm{O}_{29}$ and $(\mathrm{Mo}, \mathrm{W})_{11} \mathrm{O}_{32}$.

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[^0]:    * A survey of this investigation was presented at the meeting of the American Crystallographic Association in Camp Tamiment, Pa., June 1952.

